

An Efficient Network Coding based Retransmission Algorithm for Wireless Multicast

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Abstract—Retransmission based on packet acknowledgement (ACK/NAK) is a fundamental error control technique employed in IEEE 802.11-2007 unicast network. However the 802.11-2007 standard falls short of proposing a reliable MAC-level recovery protocol for multicast frames. In this paper we propose a latency and bandwidth efficient coding algorithm based on the principles of network coding for retransmitting lost packets in a single-hop wireless multicast network and demonstrate its effectiveness over previously proposed network coding based retransmission algorithms.

I. INTRODUCTION

One-to-many (broadcast/multicast) transmission scheme is popular for many applications, and is widely implemented in Wireless Local Area Networks (WLANs) for its effectiveness in bandwidth consumption in a spectrum-limited wireless space. WLANs transmission is currently dictated by standards set out by IEEE 802.11-2007 [1]. For one-to-one (unicast) wireless transmission, transmission reliability is achieved through Automatic Repeat Request (ARQ) variants or/and Forward Error Correction (FEC) schemes. Since broadcast is a special case of multicast, without loss of generality we will use the term multicast henceforth. However for a multicast, no consideration is made for ACK/NAK and RTS/CTS packet exchange in 802.11-2007 except for those frames sent with the To DS field set. Additionally, for multicast network where consideration for control packet is made, such packets are collected individually one-by-one, and so is the retransmission of the lost packets done, that is, one-by-one. As such for multicast network, the reliability problem is two-folded: 1) Efficient mechanism for the transmission of control packets (ACK/NAK, RTS/CTS), and 2) efficient retransmission of packets lost.

As multicasting is gaining popularity for applications such as file distribution and multimedia conferencing, a more reliable scheme is needed for the fulfillment of future growth in multicast network. Motivated by promising applications of Network Coding (NC), recent works [2] - [5] have demonstrated the suitability of NC for retransmission of lost packets to improve bandwidth performance in a multicast network.

Our algorithm is based on the concept of network coding [9], [10]. Network coding in its simplest form exploits the fact that rather than transmitting wireless packets individually to some receivers which may be ‘overheard’ by some other receivers already having those packets, and vice versa, it is

often possible to combine those packets using bit-by-bit XOR (denoted by \oplus) and transmit it as a single coded packet, which can then be decoded by all (/most) of the receivers based on the packets they already have. For illustration consider that receiver R_1 has packet c_1 but not c_2 , while R_2 has c_2 but not c_1 . Rather than transmitting these two packets individually, the transmitter can encode c_1 and c_2 to generate $c_1 \oplus c_2$, which is then multicast to both the receivers and decoded.

The remaining paper is organized as follow: In Section II we give an overview of related work, followed by the problem statement in Section III. Following that, we discuss previously proposed coding algorithm in Section IV and our BENEFIT algorithm in Section V. We then confirm the performance of BENEFIT with simulation results in Section VI, and finally present conclusion in Section VII.

II. RELATED WORK

Packet retransmission based on network coding for a one-to-many, single-hop multicast network is a recent field of study, first proposed by D. Nguyen et al. [3], which was later further elaborated into [4] by D. Nguyen et al. In [4] the authors demonstrate bandwidth effectiveness achieved by employing greedy network coding for retransmission over traditional ARQ schemes through simulation work. In [2] the authors follow up the work in [3] by comparing various packet coding algorithms for packet retransmissions. While in [6], the authors presents an analytical work on the reliability performance of network coding compared with ARQ and FEC in a lossy network. Network Coded Piggy Back (NCPB) [7] demonstrates an efficient and practical testbed implemented random linear network coding based many-to-many reliable network model for real-time multi-player game network.

Since our work primarily focuses on proposing an efficient network coding based retransmission algorithm for a one-to-many single-hop network, we will be comparing our results with the algorithm given in [2] which is the most closely related work.

A. Our Contribution

The novelty of our work is the development of a computationally feasible network coding based retransmission algorithm whose gains are two-folded: 1) Our algorithm BENEFIT delivers better throughput with respect to the current best single-hop, NC based retransmission algorithm, and 2) we

also demonstrate that our algorithm achieves minimum time to decode packets. None of the previous works [2] - [5] on NC based retransmission incorporates consideration of packet latency in their work.

Here we will also show that it is no longer necessary to follow the packet coding rule [10], [11] strictly. This relaxation in the coding rule has the potential for modification and development of other network coding based applications.

III. PROBLEM STATEMENT

Consider a single-hop multicast network with M fixed receiver stations R_i , (i is the receiver station ID, $1 \leq i \leq M$) with static membership and $M \geq 2$, and a single transmitting station T_x . Packet batch size is denoted by N . Packet reception at R_i follows Bernoulli model, whereby a successful reception of packet c_k (k is the datagram packet ID, $1 \leq k \leq N$) at R_i is indicated by '0' and packet loss by '1' in the transmission matrix (see Table I). A transmission matrix is a 2-dimensional array table, where the rows represents R_i and columns represents c_k . For a given packet, its packet utility cu_k ($0 \leq cu_k \leq M$) is defined as the number of receiver(s) which have not received the packet (i.e. the numbers of '1's in a given column). While the receiver utility ru_i indicates the number of packet(s) not received by the receiver R_i from a given set of specified packet(s). For Bernoulli model, packet loss at all receivers is homogeneous, and is determined by a fixed loss probability p_i , which gives a packet successful reception probability of $1 - p_i$. For a fixed batch size, L_i denotes the number of lost packets for station i . Q_j is the probability that after N transmissions, the total number of packets lost is no more than j ($1 \leq j \leq N$).

The time taken for one transmission is represented by one time slot. The *time to decode* a lost packet for a given R_i is the total number of transmissions (original transmission, retransmission and transmission of coded packet) after which the lost packet is recovered by the given R_i .

For simplicity we assume that there is a reliable control packet exchange mechanism in the network¹ and that all coded/retransmitted packets are successfully received by the receivers. In the context of BENEFIT algorithm, a *benefit* value is generally defined as the number of '1's in the transmission matrix which are converted to '0's after the transmission of a coded packet or retransmission of the packet, hence the name of the algorithm: 'BENEFIT'.

A. Theoretical Numbers of Retransmissions

The probability that $L_i \leq j$, for a single R_i is given by

$$P[L_i \leq j] = \sum_{c=0}^j \binom{N}{c} p_i^c (1 - p_i)^{N-c}. \quad (1)$$

¹Control packets in multicast network can be implemented by designing ACK packets from multiple STA such that, upon reception of these simultaneously transmitted ACK packets, the original sender is able to efficiently decode the packet which is the superimposition of all ACK packets and infer which receiver STA have received the datagram packet [8].

The probability that all M stations experience a packet loss rate no more than j is given by $\prod_{i=1}^M P[L_i \leq j]$. Given this result, the probability that the total number retransmission is j , is given by

$$Q_j = \prod_{i=1}^M P[L_i \leq j] - \prod_{i=1}^M P[L_i \leq j-1]. \quad (2)$$

A more elaborative discussion of retransmission bandwidth for different transmission schemes compared with network coding is given in [4], [6].

IV. CODING ALGORITHM

Previous coding algorithms (except random linear network coding, RLNC) were build on the foundation of a simple *packet coding rule* [10], [11]:

For T_x to transmit (/retransmit) M packets c_1, \dots, c_M to M receivers, R_1, \dots, R_M respectively, the coded packet obtained by coding M packets c_1, \dots, c_M can only be decoded at R_i if R_i has $(M-1)$ of c_j packets, except c_i ($j \neq i$).

We now discuss the major coding algorithm used in network coding literature.

A. Greedy Network Coding

A greedy algorithm for coding packets has been traditionally used in several network coding based literature and still continues to be a dominant approach in many NC networks like IP-level routers in the Internet [9], wireless mesh network [10] and multi-hop wireless routing [11]. A greedy network coding algorithm, like traditional greedy algorithm makes coding decisions which gives optimal local results. A greedy coding algorithm encodes current locally available packets iteratively as long as it can be decoded by all the intended receivers, without consideration whether its a 'globally' optimal solution or not.

B. Random Linear Network Coding

RLNC [7] is a decentralized network coding approach, whereby the coded packet is given by $c_{coded} = \sum_{k=1}^{k=N} g(e)c_k$, where $g(e)$ is the global encoding vector, and is included in c_{coded} as an overhead information in the packet header. Each of the receiver R_i must successfully receive N innovative packets (i.e. coded packets which are linearly independent of the previously received coded packet). Once the receivers have N innovative packet, it can then decode N packets using simple matrix inversion.

C. Sort-by-Utility

The coding algorithm which E. Rozner et. al. [2] proclaims to be the delivering the best performance for a one-hop multicast network is Sort-by-Utility. Therefore we will be comparing our BENEFIT algorithm with Sort-by-Utility for evaluation purposes. In a Sort-by-Utility coding algorithm, the T_x first transmits N packets, and then sorts the packets in descending order of their cu_k values, using arrival time as tie-breaker for those packets have equal packet utility. Once

the packets are sorted, the remaining operation of Sort-by-Utility is essentially a greedy coding algorithm, i.e. the T_x then iteratively starts coding successive packets starting from packets having highest packet utilities and codes them with successive sorted packets as long as the coded packet can be decoded by *all* receivers.

TABLE I
TRANSMISSION MATRIX EXAMPLE

R_i/c_k	ru_i	c_1	c_2	c_3	c_4	c_5
cu_k	10	2	3	1	2	2
R_1	3	1	1	0	0	1
R_2	2	0	1	0	1	0
R_3	2	0	1	1	0	0
R_4	3	1	0	0	1	1

Consider as an example the matrix given in Table I. Sort-by-Utility algorithm after initial packet transmission (c_1 to c_5) would sort the transmitted packets based on its packet utilities cu_k , i.e. c_2, c_1, c_4, c_5, c_3 . Then T_x transmits the packets as follows: $c_2, c_1 \oplus c_3, c_4, c_5$. Thus requiring a total of 4 retransmissions with an average *time to decode* a packet to be 4.4 time slots.

V. BENEFIT ALGORITHM

BENEFIT (Fig 1), unlike Sort-by-Utility does not need to wait until the end of the batch size (N packet transmissions) before starting the retransmission process. It start transmitting coded packet once the prospective coding packets satisfy the following three conditions: *CodingBenefit()*, *ColumnsBenefit()* and *CombinationBenefit()* (see Table II). Retransmitting as soon as the right conditions are met rather than wait till the end of the batch size in effect reduces the time to decode the packet. BENEFIT works on the basis that *it is not necessary for the coded packet to be decodable by all the receivers immediately*, assuming that the non-decodable coded packet can be decoded based on future transmission of coded packet(s). This principle is in essence the key strength of BENEFIT and thus, this way it contrasts the traditional *packet coding rule*.

For the first scan cycle, to decide whether to transmit a packet c_k or to scan the next packet (see the first step of Fig. 1) is decided based on the fact that if any previously transmitted packet has never been the first prospective coding packet (i.e. $pros_pks[0]$) in that cycle, and the *current* value of cu_k of that previously transmitted packet is $1 \leq cu_k < M$, then the algorithm scan the next packet and stores it as the first prospective coding packet, else it transmits the next packet. For consecutive cycles, the algorithm only scans the packets. *CodingBenefit()* ensures that packets are only coded, if the immediate benefit derived from such coding outweighs or equals the benefit derived from transmitting a single packet (uncoded) with minimum packet utility (cu_k) from the set of prospective coding packets and considered packet (c_k). *ColumnsBenefit()* selects the most suitable (/fittest) packets for coding, by eliminating those packets which can not be

TABLE II
DEFINITION OF BENEFIT TERMS

<i>CodingBenefit()</i> Checks the following equality: $DecodeBenefit() \geq MinimumBenefit()$
<i>DecodeBenefit()</i> Finds out how many receivers STA will benefit <i>immediately</i> from the transmission of the coded packet, $0 \leq DecodeBenefit() \leq M$.
<i>MinimumBenefit()</i> From the list of prospective coding packets, find c_k with minimum current packet utility cu_k , $0 \leq MinimumBenefit() \leq M$.
<i>ColumnsBenefit()</i> From the list of prospective coding packets, checks if every packet can be decoded by at least one receiver STA immediately.
<i>CombinationBenefit()</i> From the list of considered packets for coding, calculates the number of receiver STA which will benefit either <i>immediately</i> ($ru_i=1$) or in <i>future</i> ($ru_i \geq 2$) from the coding of the prospective coding packets. And then checks if its equal to <i>DesiredBenefit</i> (see Fig 1). ru_i value in the context of <i>CombinationBenefit()</i> is computed only for the packets in $pros_pks[]$, $CombinationBenefit() \geq DecodeBenefit()$.
<i>DecodeSearch()</i> Decodes the arrived coded packet if it can. If the packet gets decoded, then search the memory for any previously non-decodable packet which can now be decoded based on the current decoded packet, and decode it.
<i>Benefit immediately/ Immediate decoding</i> The decoding of coded packet 'on the spot,' without the need for information from future transmission(s).

decoded by at least one receiver STA immediately. If the packets satisfy *CodingBenefit()* and *ColumnsBenefit()* conditions but not *CombinationBenefit()*, then they are considered eligible prospective coding packets for combination with other packet(s), and thus the algorithm then searches (scans) for other packet(s), which in combination with the previous prospective coding packets will satisfy the three conditions for packet coding. If the algorithm reaches the end of the batch size and there are still '1's in the transmission matrix, then the *CombinationBenefit()* condition is relaxed by decrementing *DesiredBenefit* and the algorithm then starts a new scan cycle (maximum of $M-1$ scan cycles) until the transmission matrix is composed of $M*N$ '0's.

A. Computational Complexity of BENEFIT

The computational complexity of *CodingBenefit()*, *ColumnsBenefit()* and *CombinationBenefit()* all grow linearly with respect to the number of prospective coding packets and receivers. While *DecodeSearch()* can be implemented using a binary search algorithm whose average complexity is logarithmic. Given that the number of prospective coding packets increases with the number of receivers, the computational complexity of BENEFIT can be considered to be linearly increasing with the number of receiver STAs.

B. Illustrative example - BENEFIT

Consider the algorithm given in Fig. 1, illustrated with the transmission matrix given in Table I. After the T_x transmit c_1 , c_1 is stored as the first prospective coding packet. T_x then transmit c_2 , c_1 and c_2 are then checked for *CodingBenefit()* and *ColumnsBenefit()* conditions, which they satisfy. Hence they are then checked

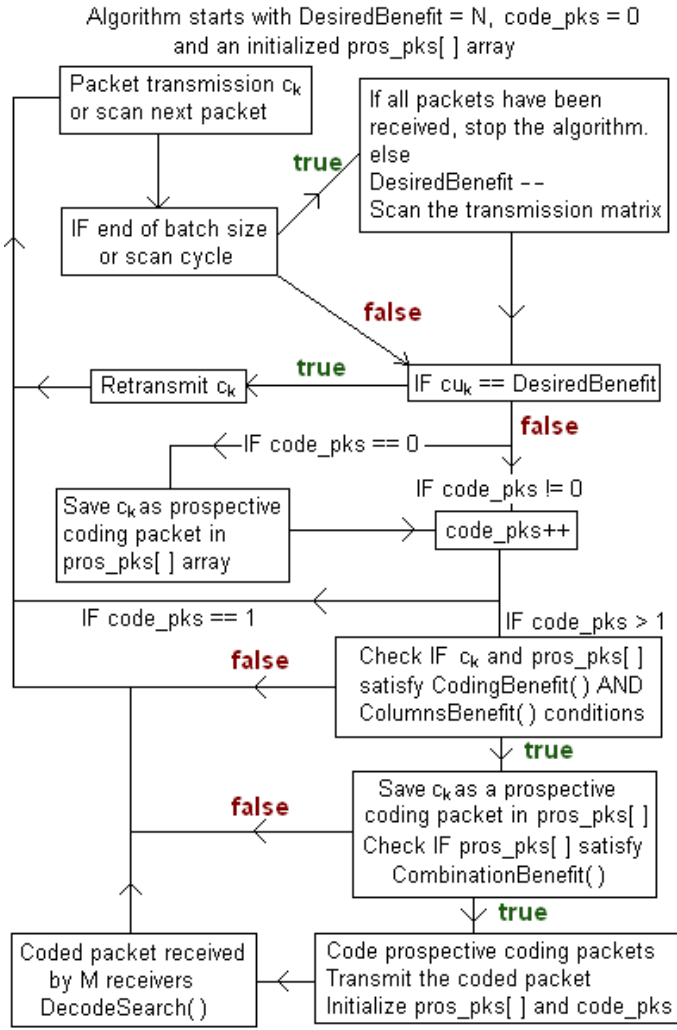


Fig. 1. BENEFIT algorithm - flowchart

for *CombinationBenefit()* condition. Since both c_1 and c_2 satisfy the *CombinationBenefit()* condition as well, the packets are coded and transmitted. Only R_1 is not able to decode $c_1 \oplus c_2$ immediately (current value of $cu_1 = cu_2 = 1$). The algorithm then scan the next packet c_2 and stores it as the first prospective coding packet, and the T_x then transmit c_3 . Since c_2 and c_3 satisfy *CodingBenefit()* and *ColumnsBenefit()* conditions but not *CombinationBenefit()* condition, c_3 is therefore saved as a prospective coding packet. The T_x then transmits c_4 , which in addition to c_2 and c_3 satisfy *CombinationBenefit()* condition. Hence the packets are coded and transmitted. All receivers are able to immediately benefit from the transmission of $c_2 \oplus c_3 \oplus c_4$. *DecodeSearch()* function at R_1 after decoding $c_2 \oplus c_3 \oplus c_4$, decodes $c_1 \oplus c_2$ using c_2 and obtains c_1 . The last packet in the batch c_5 is then transmitted and stored as prospective coding packet, however as it is obvious, there is not any possibility of finding coding packets for c_5 , the algorithm decrements the value of *DesiredBenefit* twice, following which c_5 is retransmitted

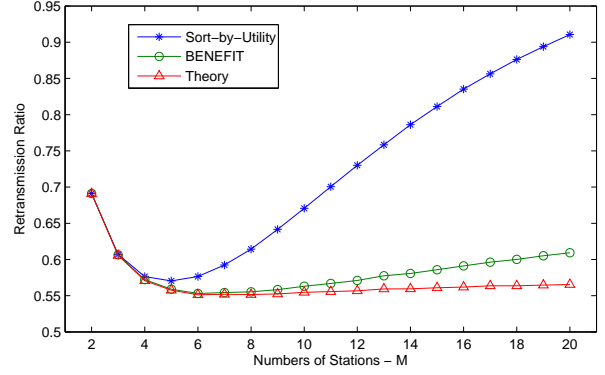


Fig. 2. Retransmission ratio against M , for $p_i=0.5$, and $N=200$

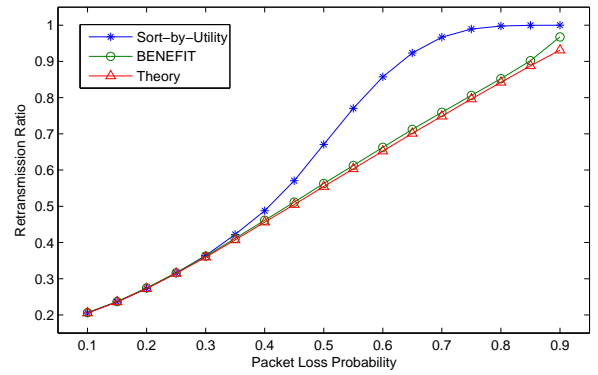


Fig. 3. Retransmission ratio against p_i , for $M=10$, and $N=200$

without any encoding in the third scan cycle.

This example illustrates that BENEFIT requires a total of only 3 retransmissions ($c_1 \oplus c_2$, $c_2 \oplus c_3 \oplus c_4$ and c_5) in contrast to 4 retransmissions used by Sort-by-Utility (see section IV-C), with an average *time to decode* a packet of 1.9 time slots ($c_1 \oplus c_2$ transmitted after the transmission of c_2 , and $c_2 \oplus c_3 \oplus c_4$ transmitted after the transmission of c_4) in contrast to 4.4 time slots used by Sort-by-Utility. Hence for this example, it has been shown, that BENEFIT outperforms Sort-by-Utility both in terms of retransmission bandwidth and packet delay.

VI. SIMULATION RESULTS

We construct a C++ based discrete time simulator, using Random Number Generator to generate transmission table like the one given in Table I. The characteristics of the network shall be the same as mentioned in Section III. For each set of values, the simulation is repeated 1000 times. For performance evaluation, we use *retransmission ratio* (also used in [2]) which is defined as the total number of retransmissions using coding algorithm divided by the total number of retransmissions using traditional 802.11 retransmission scheme. Theory in Fig. 2 and 3 refers to retransmission ratio obtained by dividing Q_j (derived in Section III-A) with the total number of retransmissions using traditional 802.11 retransmission

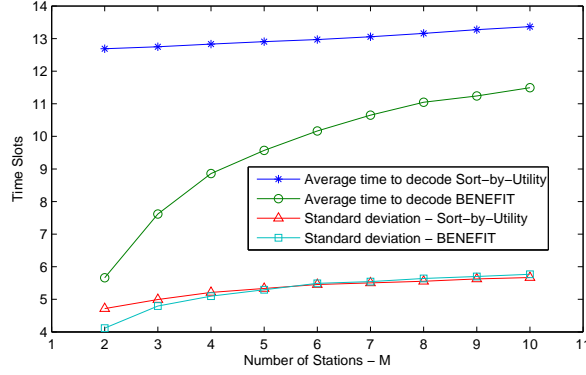


Fig. 4. Time slots against M , for $p_i=0.25$, and $N=20$

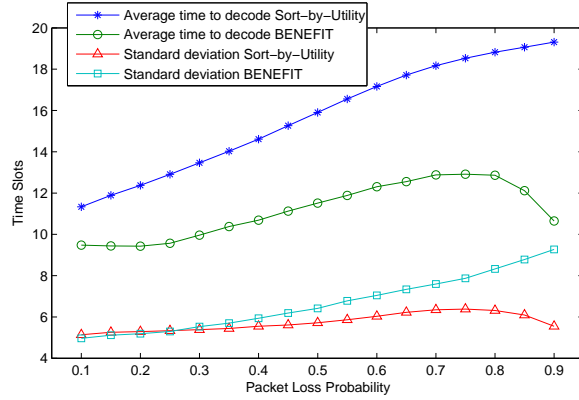


Fig. 5. Time slots against p_i , for $M=5$, and $N=20$

scheme.

Figure 2 shows that BENEFIT consistently performs better than Sort-by-Utility. The initial trough in the graph is because of more coding opportunity available with an increase in number of STA. A simple heuristic explanation for this is that for 2 STA, there is scope for only 2 packets to be coded, however for 4 STA, there is scope for 2, 3 and 4 packets to be coded together based on opportunities. However as the number of STA increases further, retransmission ratio starts increasing as then, increase in conflict opportunities (as shown in Fig. 4, as the number of STA increases, the time to search for prospective coding packet also increases) between coding packets outweighs increase in coding opportunities. Figure 3 shows the performance of BENEFIT over a range of loss probability values. Figure 2 and 3 proves that for a small-medium network BENEFIT performs close to the theoretical bound for all ranges of p_i .

While the bandwidth performance of BENEFIT and Sort-by-Utility is almost similar for low loss probability and/or small network, for such networks BENEFIT can still be useful for real-time applications which are highly delay sensitive. As Fig. 4 shows that even for a small batch size, the average time BENEFIT takes to decode/retransmit a

packet is far less than that of Sort-by-Utility. BENEFIT latency efficiency can be improved further by decrementing the initial value of *DesiredBenefit*, which will relax the *CombinationBenefit()* condition and thus require the T_x to spend less time searching for suitable coding packet. Decreasing the batch size also reduces packet retransmission delay as show in [2], [4]. However both these techniques will come at a tradeoff cost of an increase in retransmission ratio. Flexibility to balance throughput-delay tradeoff in BENEFIT allows the network designer to modify the algorithm based on the network requirements. Figure 5 shows that the average time to decode packet gradually increases with p_i for BENEFIT, however as p_i crosses 0.8 the average time to decode packet starts decreasing as most of the packets satisfy $cu_k == \text{DesiredBenefit}$ condition and hence are retransmitted without any coding. The time saved searching for prospective coding packets reduces the average time to decode. This also explain an increase in standard deviation for BENEFIT in Fig. 5, as some packets are retransmitted without any encoding (shorter waiting time), while other packets need to wait for longer to find suitable coding partners.

VII. CONCLUSION

In this paper we have demonstrated a computationally feasible, bandwidth and latency efficient retransmission coding algorithm, which doesn't strictly follow the traditional coding rule. Selectively modifying the BENEFIT conditions would also allow the network designer to adjust the algorithm as per the throughput-delay requirements of the network. We believe that there is potential for research work to exploit relaxation in the coding rule, and study modifications to mechanisms like COPE [10] based on rules derived from BENEFIT.

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